Digital Imaging Study Group July 31, 2003

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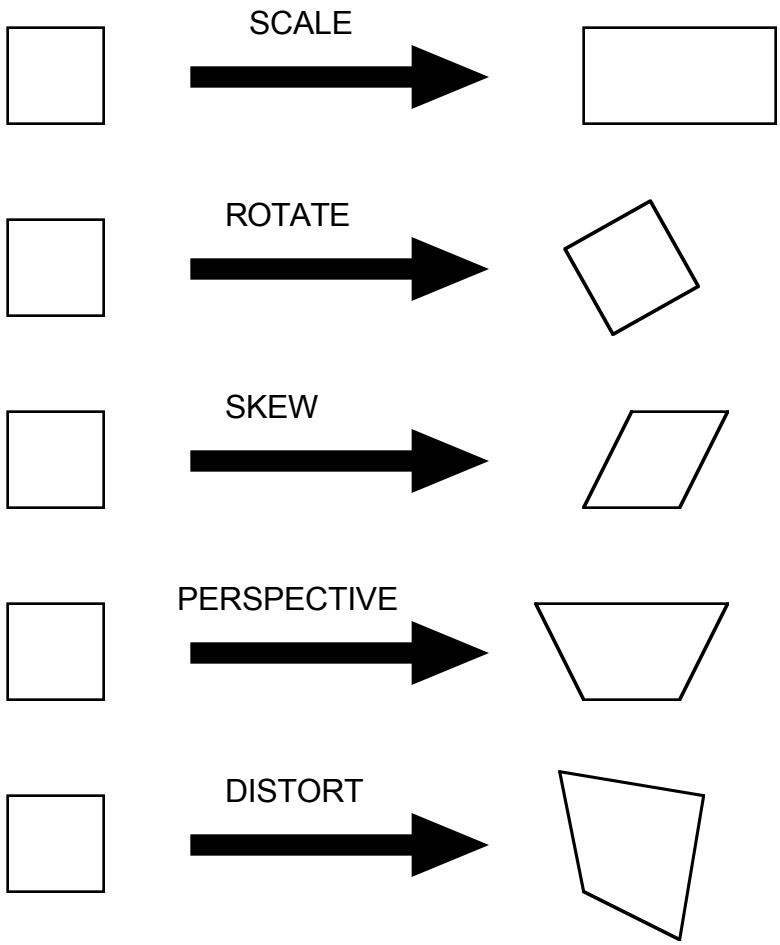
Transformations and Perspective

The Transform command (under the Edit menu) geometrically maps a selection or layer into a new shape. It includes Scale, Rotate, Skew, Perspective, and Distort:



Scale is useful when combining elements from two or more photographs which may not have the appropriate relative size. Rotate is good for straightening horizons. And Perspective is nice for correcting perspective distortion (discussed below).

Transform operates on a Selection or a Layer (or group of linked layers). To Transform the entire Background layer, either Select>All, or Convert the Background to a Layer (by double-clicking it in the Layers Palette).

The transformed pixels will not line up with the pixels in the underlying canvas, so interpolation is required. Choose the type (bicubic, etc.) in Preferences. To avoid multiple interpolations, do several transformations at once. Once you choose the Transform command, you can do do as many transformations of any type as you like. You see the previews, but they are not actually applied until you hit Enter or Return.

Control the Transform by dragging the handles which appear on the corners and sides of a rectangular box enclosing your selection or layer. (You can also enter numerical values (percentages, angles, etc.) in the options bar.)

You are likely to drag parts of the selection or layer outside the canvas, and/or to leave gaps where the transformed selection or layer no longer covers the canvas. (You could enlarge the canvas in advance to capture more of the image.) When the transformation is applied, the parts outside the canvas are lost, and the gaps are transparent (white background if there is nothing else behind it). You may need to re-crop afterwards.

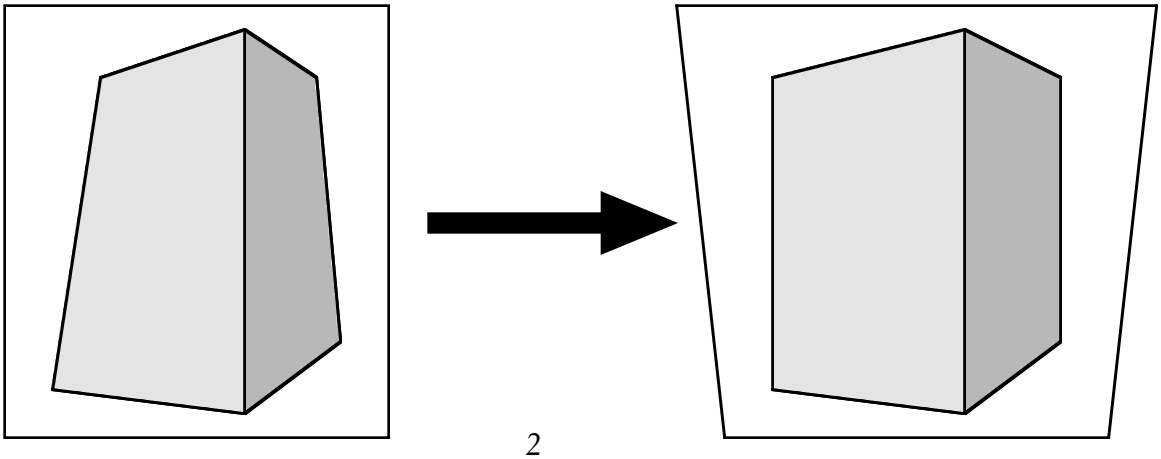
You will probably want to drag some of the control handles outside the canvas, so you will need room to work. Make the window larger than the canvas. You can do this by dragging the corner of the window to make it larger, or by shrinking the canvas by option-clicking with the Zoom Tool. (While a transform is being previewed, you can't choose tools, but you can do the same thing using Command-option-space, etc.)

It helps to have some horizontal and vertical reference lines to check how your image is lining up. Create guidelines with View>New Guide, or by dragging with any tool from the rulers into the image. Reposition guides using the Move Tool. Get rid of a guide by dragging it out of the window.

Perspective Distortion

Perspective Distortion is the phenomenon where lines which are parallel in the three-dimensional world are not parallel in a two-dimensional representation. It is not a lens flaw, but an inherent result of mapping the 3-D universe onto film or paper. It is not necessarily bad, and can create a dramatic effect. But it is sometimes esthetically displeasing when lines we know are supposed to be vertical or horizontal are not parallel to the vertical or horizontal edges of a rectangular image.

With an ideal camera and lens, lines is the real world which are parallel to the film plane will be parallel on the image. So if the camera is held level, not tilted right or left or up or down, then vertical line in the real world should be vertical on the image. But if the camera is tilted upwards, for example to take a picture of a nearby tall building, then the vertical lines of the building will tilt inwards on the film. A Perspective Transformation can straighten them out:



If the photo is taken with the camera pointed squarely at the side of the building, so that the film plane is parallel to the building facade, then the horizontal lines in the building facade should also be horizontal on the film. If it is slightly off, a perspective transform can fix this too.

But consider just the vertical lines. Vertical lines in the center of the image should be vertical even with the camera tilted up. If they are not, it means that the camera was also tilted right or left. This can be corrected with a Rotate Transform.

If the camera was tilted up, the vertical lines will slant inward toward the top of the picture. You can correct this in a Perspective Transform by stretching the top corners outward, the bottom corners inward, or a combination of these. But note that if you drag the top corners out, it will tend to make the image short and fat, while if you drag the bottom corners in it becomes tall and skinny. Which is correct?

It turns out that to get the vertical and horizontal dimensions in the correct proportions, you need some more information. Sometimes you know something about actual objects in the photo, for example that some object is exactly as wide as it is high. In that case, you can use the Scale Transform to stretch the height or width to make it right. Alternatively, if you know the focal length of the lens used to take the picture, you can compute the answer and apply it. (The mathematical formula on page 4 of these notes.)

Step-by-Step Details on Correcting Vertical Perspective Distortion

- Select the entire Background or convert it to a layer. Make the window as large as possible, but make the image somewhat smaller so that you have some empty space around the image to work in when dragging the Transform handles. Create some vertical Guides near the prominent vertical lines in the image.

- Vertical lines in the center of the image should already be vertical. If not, correct this with Edit>Transform>Rotate, but don't apply the transformation yet.

- Select Edit>Transform>Perspective and make all the vertical lines truly vertical by dragging the corners horizontally. Drag the top corners outward and/or the bottom corners inward. Don't worry about the horizontal/vertical proportions now; just get the vertical lines vertical. (Caution: If your lens has pincushion or barrel distortion, the vertical lines become curved, making it impossible to get them all vertical. Look for vertical lines that extend equally above and below the middle of the image, and try to align them to vertical as much as possible along their overall length. Or go to www.dpreview.com/learn and read their article on correcting barrel distortion.) Don't apply the transformation yet.

- Select Edit>Transform>Scale and option-drag (alt-drag in Windows) the side handles to adjust the width of the trapezoid so that the horizontal and vertical proportions are correct. You can use either the known proportions of some object in the picture, or the calculated ratio of midpoint width to height, from the formula below. But don't apply the transform yet.

• Parts of the trapezoid may lie outside the canvas, and parts of the canvas may not be covered. Shift-option-drag (shift-alt-drag in Windows) a corner handle to proportionally scale the trapezoid so the the important parts of the image are as large as possible while still able to fit on the canvas. Drag anywhere inside the trapezoid to reposition it as needed over the canvas. Now apply the transform by hitting Enter.

• If parts of the canvas are empty, re-crop (or fill by cloning) as needed.

Mathematical Formula for Correct Width/Height Proportions

Here is the formula for calculating the correct image width (see illustration below). (The derivation of this formula is in the appendix, posted on the Club website.)

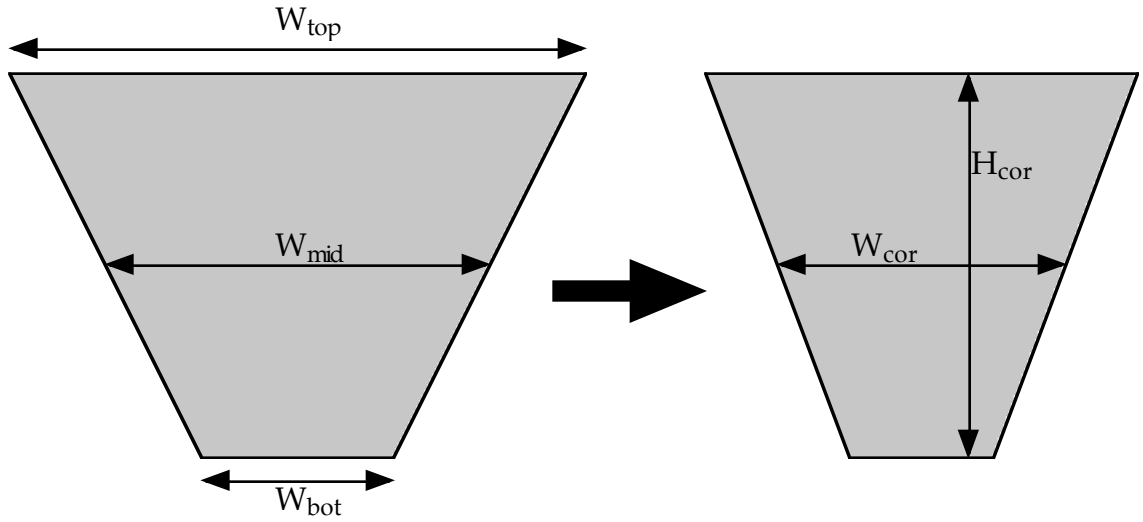
• To make the vertical lines truly vertical, you will have transformed the rectangular image into a trapezoidal shape with the top wider than the bottom. Measure the width at the top and bottom, and call them W_{top} and W_{bot}. The width at the midpoint is $W_{mid}=(W_{top}+W_{bot})/2$. Calculate the ratio $R=(W_{top}-W_{bot})/W_{mid}$.

• Determine the focal length of the lens used to take the photo, and call it F. Determine the width and height of the film (or digital sensor) used to capture the image, and call them W_{film} and H_{film}. For 35 mm film, W_{film} would be 36 mm and H_{film} 24 mm in landscape orientation, and vice versa in portrait. For information on the actual sensor dimensions of digital cameras, see:
http://www.dpreview.com/news/0210/02100402sensorsizes.asp.

• We assume that the image covers the whole film or sensor frame. The correct ratio of midpoint width to height is:

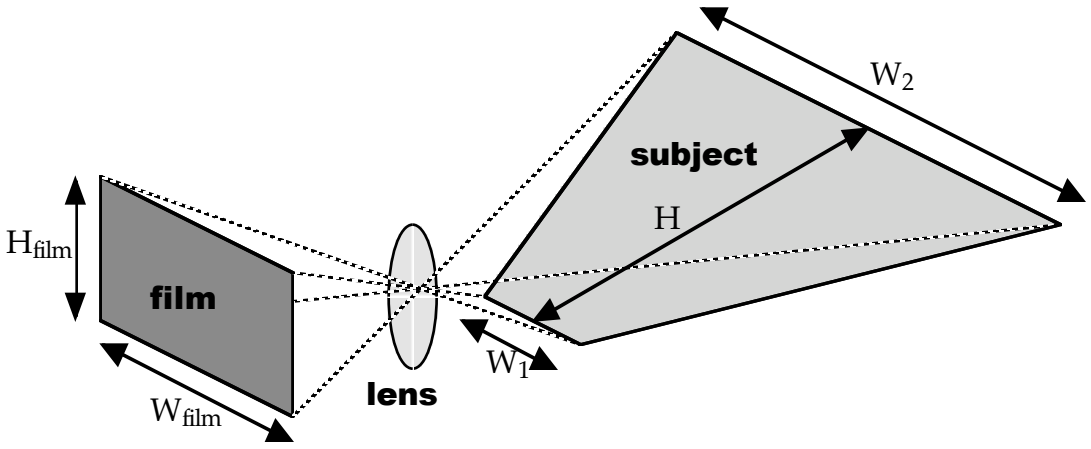
$$W_{cor}/H_{cor} = W_{film} \Big/ \sqrt{\left(H_{film}{}^2+R^2F^2\right)}$$

For correct proportions, scale the trapezoid accordingly.

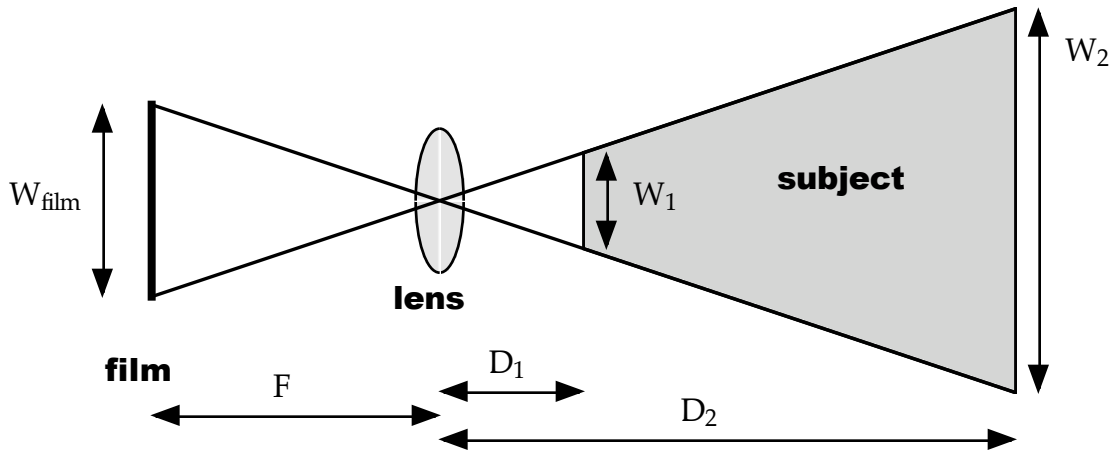


Appendix: Derivation of Formula for Correct Width/Height Proportions

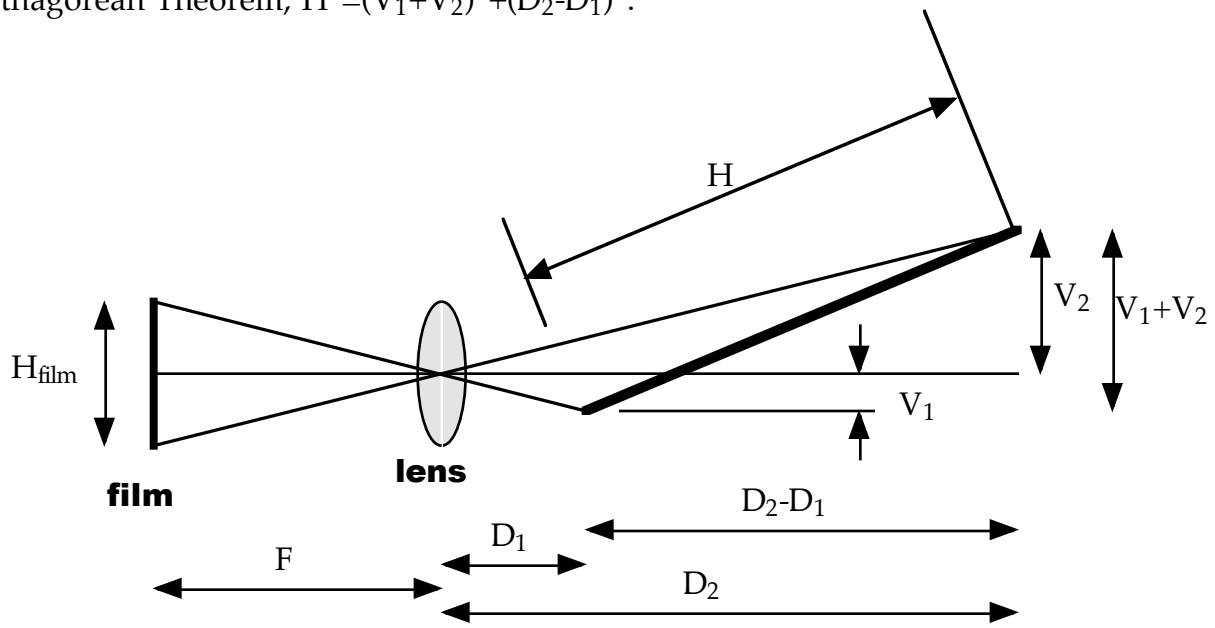
To derive this formula, consider the geometry of the photographic situation shown below. Although we usually think about the camera being tilted up, we are going to draw this as if the camera were level and the subject on a slope. The rectangular film frame captures the image of a trapezoidal section of the subject. The film is W_{film} wide and H_{film} high. The subject area is W_1 wide at the bottom and W_2 wide at the top. The height of the subject, measured in the plane of the subject, is H.



Now consider a top view, illustrated below. The horizontal distance of the film from the lens is the focal length, F, (assuming that the subject is much farther from the lens than the film, which is normally the case except for extreme close-ups). The horizontal distance from the lens to the bottom of the subject is D_1 and to the top D_2. Because of similar triangles, we can show that $D_1=FW_1/W_{film}$, and $D_2=FW_2/W_{film}$.



Next consider a side view shown below. The vertical distance from the midpoint of the film and lens to the bottom of the subject area is V_1, and to the top V_2. Because of similar triangles, we can show that $V_1=(D_1/F)(H_{film}/2)$, and $V_2=(D_2/F)(H_{film}/2)$. By the Pythagorean Theorem, $H^2=(V_1+V_2)^2+(D_2-D_1)^2$.



Combining the above equations gives: $H^2W_{film}^2=H_{film}^2(W_1+W_2)^2/4+F^2(W_2-W_1)^2$.

Now consider the image in Photoshop, as illustrated on page 4. To correct perspective, the image has been transformed so that it is W_{top} wide at the top and W_{bot} at the bottom. The image is proportional to the subject, except that the scale may be different in the horizontal dimension than in the vertical dimension, so $W_{top}/W_2=W_{bot}/W_1$. We have defined $W_{mid}=(W_{top}+W_{bot})/2$, and $R=(W_{top}-W_{bot})/W_{mid}$. If we also define $W_3=(W_1+W_2)/2$, it is easy to show that $R=(W_2-W_1)/W_3$. For correct proportions, we want $W_{cor}/H_{cor}=W_3/H$. We can combine the above equations to show that:

$W_{film}^2H_{cor}^2/W_{cor}^2=H_{film}^2+R^2F^2$. Therefore $W_{cor}/H_{cor}=W_{film}/\sqrt{(H_{film}^2+R^2F^2)}$.

Q.E.D.